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The confining heterotic brane gas: a non-inflationary solution to the entropy and horizon problems of standard cosmology

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ABSTRACT: We propose a mechanism for solving the horizon and entropy problems of standard cosmology which does not make use of cosmological inflation. Crucial ingredients of our scenario are brane gases, extra dimensions, and a confining potential due to string gas effects which becomes dominant at string-scale brane separations. The initial conditions are taken to be a statistically homogeneous and isotropic hot brane gas in a space in which all spatial dimensions are of string scale. The extra dimensions which end up as the internal ones are orbifolded. The hot brane gas leads to an initial phase (Phase 1) of isotropic expansion. Once the bulk energy density has decreased sufficiently, a weak confining potential between the two orbifold fixed planes begins to dominate, leading to a contraction of the extra spatial dimensions (Phase 2). String modes which contain momentum about the dimensions perpendicular to the orbifold fixed planes provide a repulsive potential which prevents the two orbifold fixed planes from colliding. The radii of the extra dimensions stabilize, and thereafter our three spatial dimensions expand as in standard cosmology. The energy density after the stabilization of the extra dimensions is of string scale, whereas the spatial volume has greatly increased during Phases 1 and 2, thus leading to a non-inflationary solution of the horizon and entropy problems.

KEYWORDS: p-branes, Cosmology of Theories beyond the SM.

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1. Introduction

The Inflationary Universe scenario [1] (see also [2-4]) has been extremely successful phenomenologically. It has provided a solution to some of the key problems of standard cosmology, namely the horizon and flatness problems, and yielded a mechanism for producing primordial cosmological perturbations using causal physics, a mechanism which predicted [5, 6] (see also [7, 2]) an almost scale-invariant spectrum of adiabatic cosmological fluctuations, a prediction confirmed more than a decade later to high precision by cosmic microwave background anisotropy experiments [8–11].

In this paper, we will pay special attention to the "entropy problem" of standard cosmology [1]. The problem consists of the fact that without accelerated expansion of space, it is not possible to explain the large entropy, size and age of our current universe without assuming that at very early times the universe was many orders of magnitude larger than would be expected on dimensional arguments.

In the inflationary scenario, the entropy problem is solved by postulating a sufficiently long period of accelerated expansion, after which the universe reheats to a temperature comparable to that prior to the onset of the period of acceleration. In most] models of inflation, the accelerated expansion of space is sourced by the potential energy of a slowly rolling scalar field. Such models, however, are subject to serious conceptual problems (see e.g. [12, 13] for recent overviews of these problems). Most importantly, the source of the acceleration is very closely related to the source of the cosmological constant in field theory, a constant which is between 60 and 120 orders of magnitude larger than the maximal value of the cosmological constant allowed by current observations. Because of the existence of these conceptual problems, it is of great importance to look for possible alternatives to scalar field-driven inflationary cosmology.

There have been various suggestions for alternative cosmologies. In varying speed of light models [14, 15], postulating the existence of a period in the early universe during which the speed of light decreased very fast leads to a solution of the horizon problem. In the "Pre-Big-Bang scenario" [16], the Universe is born cold, flat and large, undergoes a period of super-exponential contraction before emerging into the period of radiationdominated expansion of standard cosmology. The contracting phase and the expanding phase are related via a duality of string theory, namely "scale-factor duality". In a more recent cosmological scenario motivated by heterotic M-theory [17], namely the "Ekpyrotic scenario" [18], the collision of a bulk brane onto our boundary orbifold fixed plane generates a non-singular expansion of our brane. However, neither the Pre-Big-Bang nor the original Ekpyrotic scenario can explain why our Universe is so large and old (without assuming that the Universe is already much larger than would be expected by dimensional arguments at the end of the phase of contraction (see e.g. [19, 20]) (this problem is avoided in the "cyclic scenario" [21], a further development of ideas underlying the Ekpyrotic scenario, but this is achived at the cost of additional ad hoc assumptions about the cosmological bounce). The size problem has so far also prevented the "string gas cosmology" scenario [22, 23] (see e.g. [24, 25] for recent reviews) from making contact with late time cosmology, although a stringy mechanism for producing a scale-invariant spectrum of cosmological perturbations does exist in this context [26].

In this paper, we present a potential solution of the entropy problem which does not make use of a period of accelerated expansion. Our solution makes use of several ingredients from string theory: extra spatial dimensions, the existence of branes and orbifold fixed planes as fundamental extended objects in the theory, and a stringy mechanism for stabilizing the shape and volume moduli of string theory via the production of massless string states at enhanced symmetry points in moduli space. Thus, it is possible that our mechanism will find a natural realization in string theory.

2. Overview of the model

Our starting point is a topology of space in which all but three spatial dimensions are orbifolded, and the three dimensions corresponding to our presently observed space are toroidal. Specifically, the space-time manifold is

$$\mathcal{M} = \mathcal{R} \times T^3 \times T^d / Z_2 \,, \tag{2.1}$$

where T^3 stands for the three-dimensional torus, and d is the number of extra spatial dimensions, which we will take to be either d = 6 in the case of models coming from superstring theory, or d = 7 in the case of models motivated by M-theory. We will assume that there is a weak confining force between the orbifold fixed planes.¹

¹It may be necessary to have branes pinned to the orbifold fixed planes in order to induce such a potential. Our approach, at this stage, is purely phenomenological, and we simply postulate the existence of a potential with the required properties.

As our initial conditions, we take the bulk to be filled with an isotropic² gas of branes, as in the studies of [27-29]. These studies show that, in the context of Type IIB superstring theory, the bulk of the energy density will end up in three and possibly seven branes. However, if the initial Hubble radius is large relative to the size of space, there will be no residual seven branes. In the case of heterotic string theory or taking the starting point to be M-theory, we would be dealing with Neveu-Schwarz 5-branes.

Assuming that the universe starts out small and hot, it is reasonable to assume that the energy density in the brane gas will initially be many orders of magnitude larger than the potential energy density generated by the force between the orbifold fixed planes. Thus, initially our universe will be expanding isotropically. We denote this as Phase 1. Our key observation is that in this phase, the energy density ρ_p projected onto the orbifold fixed planes, i.e. the bulk energy density integrated over the transverse directions, does not decrease. The reason is that the tension energy of the p-branes increases as $a(t)^p$, where here a(t) is the bulk scale factor. The volume parallel to the orbifold fixed planes is increasing as $a(t)^3$, and hence the projected energy density does not decrease (it is in fact constant in the case of 3-branes).

During Phase 1, the bulk energy density will decrease. Hence, eventually the interorbifold potential will begin to dominate. At this point, the cosmological evolution will cease to be isotropic: the directions parallel to the orbifold fixed planes will continue to expand while the perpendicular dimensions begin to contract. We denote this phase as Phase 2.

Once the orbifold fixed planes reach a microscopic separation, a repulsive potential due to string momentum modes becomes important (one example is the production of massless states at enhanced symmetry points [30, 31]). The interplay between this repulsive potential which dominates at small separations and the attractive potential which dominates at large distances, coupled to the expansion of the three dimensions parallel to the orbifold fixed planes, will lead to the stabilization of the radion modes at a specific radius (presumably related to the string scale). In the context of heterotic string theory, we could use the string states which are massless at the self-dual radius to obtain stabilization of the radion modes at the self-dual radius [32, 33] (see also [34]). These modes would also ensure dynamical shape moduli stabilization [35]. We denote the time of radion stabilization by t_R since this time plays a similar role to the time of reheating in inflationary cosmology. The branes decay into radiation either during or at the end of Phase 2. This brane decay is the main source of reheating of our three dimensional space.

After the radion degrees of freedom have stabilized at a microscopic value which presumably is set by the string scale, the three spatial dimensions parallel to the orbifold fixed planes will continue to expand. The energy density which determines the three-dimensional Hubble expansion rate is the projected energy density ρ_p . The key point is that during Phase 1, ρ_p does not decrease. If the bulk is dominated by 3-branes, ρ_p is constant, if it

²Note that the orbifolding will prohibit the existence of certain branes along certain of the dimensions and will thus lead to a breaking of the condition of isotropy. The details are fairly model-specific and will be discussed in a followup paper. The bottom line, however, is that the noninflationary bulk expansion of the first phase in all directions remains a valid conclusion.

is dominated by 5-branes, ρ_p in fact increases. In Phase 2, the projected energy density ρ_p also remains constant if the bulk is dominated by 3-branes, modulo the conversion of brane tension energy into radiation as the bulk branes decay or are absorbed by the fixed planes. If we approximate the evolution by assuming that all of the brane tension energy converts to radiation at the time of radion stabilization, then the value of ρ_p at t_R , which is the energy density which determines the evolution of the scale factor of our three spatial dimensions after t_R , is equal to the projected energy density at the initial time, which we take to be given by the string scale.³ If the branes decay during Phase 2, then the projected energy density at t_R is larger than the initial value, in which case we may be driven to a Hagedorn phase of string theory. The main point, however, it that since the volume of our three spatial dimensions has been expanding throughout Phases 1 and 2, the horizon and entropy problems of standard cosmology can easily be solved by simply assuming that the phase of bulk expansion lasted sufficiently long (numbers will be given later).

Note that we are assuming in this paper that the dilaton has been stabilized by some as yet unknown mechanism. In this case, the equations of motion of the bulk are those of homogeneous but anisotropic general relativity. The metric is in this case given by

$$ds^{2} = dt^{2} - a(t)^{2} d\mathbf{x}^{2} - b(t)^{2} d\mathbf{y}^{2}, \qquad (2.2)$$

where \mathbf{x} denote the three coordinates parallel to the boundary planes and \mathbf{y} denote the coordinates of the perpendicular directions. In the case of d extra spatial directions the equations of anisotropic cosmology are

$$\ddot{a} + \dot{a}(2H + d\mathcal{H}) = 8\pi Ga \left[P - \frac{1}{3+d-1}(3P + d\tilde{P}) + \frac{1}{3+d-1}\rho \right], \quad (2.3)$$

$$\ddot{b} + \dot{b} (3H + (d-1)\mathcal{H}) = 8\pi G b \left[\tilde{P} - \frac{1}{3+d-1} (3P + d\tilde{P}) + \frac{1}{3+d-1} \rho \right], \quad (2.4)$$

and

$$(3H + d\mathcal{H})^2 - 3H^2 - d\mathcal{H}^2 = 16\pi G\rho, \qquad (2.5)$$

where $H \equiv \dot{a}/a$, $\mathcal{H} \equiv \dot{b}/b$ are the expansion rates of the parallel and perpendicular dimensions, respectively, ρ is the bulk energy density and P and \tilde{P} are the parallel and perpendicular pressures, respectively.

3. The phase of bulk expansion

During the phase of bulk expansion, the two scale factors coincide, $P = \tilde{P}$, and both equations (2.3) and (2.4) reduce to

$$\frac{\ddot{a}}{a} + (2+d)\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3+d-1}[\rho - P].$$
(3.1)

 $^{^{3}}$ We assume that initial radii and densities are all set by the string scale, i.e. we introduce no unnaturally small or large numbers.

Making use of the equation of state $P = w\rho$, and inserting (2.5), the dynamical equation (3.1) becomes

$$\frac{\ddot{a}}{a} + (2+d) \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2} (3+d)(1-w) \left(\frac{\dot{a}}{a}\right)^2,$$
(3.2)

which leads to power law expansion

$$a(t) \sim t^{\alpha} \tag{3.3}$$

where the value of α depends on the equation of state:

$$\alpha = \frac{2}{(3+d)(1+w)}.$$
(3.4)

If the bulk energy is dominated by the tension of p-branes, then we have

$$w = -\frac{p}{3+d}.\tag{3.5}$$

In the example motivated by perturbative Type IIB superstring theory, namely d = 6 and p = 3 we obtain

$$a(t) \sim t^{2/(3+d-p)} = t^{1/3}.$$
 (3.6)

What is important for us is that this is not accelerated expansion. Starting with heterotic string theory, we would have d = 6 and p = 5 and for M-theory we would take d = 7 and p = 5. These two cases lead to faster expansion rates, namely $\alpha = 1/2$ in the former case and $\alpha = 2/5$ in the latter.

4. The phase of orbifold contraction

If we want the expansion which takes place in this initial phase to solve the size and horizon problems of standard cosmology independent of any further expansion during Phase 2, then the scale factor needs to increase by a factor \mathcal{F} of at least

$$\mathcal{F} \sim 10^{30} \,. \tag{4.1}$$

This result comes about by demanding that the predicted radius of the universe evaluated at the present temperature be greater than the presently observed Hubble radius, i.e. greater than 10^{42}GeV^{-1} , by taking the density at the time t_R to be given by the string scale which we take to be 10^{17}GeV , and taking into account that the scale factor in standard cosmology increases by a factor of about 10^{29} between when the temperature is of string scale and today. Correspondingly, the radiation temperature of the bulk will decrease by the same factor \mathcal{F} .

We now assume the existence of a confining potential V between the orbifold fixed planes. In order to generate such a non-vanishing potential, we will need to assume that branes are stuck to the orbifold fixed planes. In terms of the distance $r = l_s b$ between these planes (l_s being the string length), a typical confining potential is

$$V(r) = \mu r^{n} = \mu (l_{s}b)^{n}, \qquad (4.2)$$

where n is an integer, $\mu \equiv \Lambda^{d+n+4}$, and Λ is the typical energy scale of the potential. As we will show below, a value $n \geq 2\sqrt{2d} + d$ is required for our scenario to work.

The presence of this potential will lead to a transition between the phase of isotropic expansion to a phase in which the extra dimensions contract while the dimensions parallel to the fixed planes keep on expanding (and we will verify below that the expansion is not inflationary). The transition between Phase 1 and Phase 2 takes place when the bulk energy density and the inter-brane potential become comparable. The bulk energy density in Phase 1 scales as

$$p_b(t) \sim b(t)^{-d-3+p}$$
 (4.3)

(recall that in this phase a(t) = b(t)). Assuming that the initial bulk energy density is set by the string scale, and using the result (3.6), it follows that in order for the bulk to have expanded by the factor of (4.1), the upper bound on Λ should satisfy:

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$$\Lambda \sim l_s^{-1} 10^{-30\frac{d-p+3+n}{d+4+n}}.$$
(4.4)

In the case d = 6, p = 3, and n = 2 we obtain

$$\Lambda \sim l_s^{-1} 10^{-20} \sim 1 \text{MeV} \,.$$
 (4.5)

For d = 6 and p = 5 the result is $\Lambda \sim l_s^{-1} 10^{-15} \sim 100 \text{GeV}$, for d = 7 and p = 5 we obtain $\Lambda \sim l_s^{-1} 10^{-210/13} \sim 10 \text{GeV}$. Note that these values for Λ are not too different from the scale of electroweak symmetry breaking.

We will analyse the evolution during Phase 2 using a four-dimensional effective field theory, where we replace the radion b(t) by a scalar field $\varphi(t)$. In order that φ be canonically normalized when starting from the higher dimensional action of General Relativity, φ and b must be related via (see e.g. [25], appendix A)

$$\varphi = m_{pl}\sqrt{2d\log(b)}, \qquad (4.6)$$

where m_{pl} is the four-dimensional Planck mass. If the bulk size starts out at the string scale, then $b(t_i) = 1$, where t_i is the initial time. With these normalizations, $\varphi = 0$ corresponds to string separation between the branes. In terms of φ , the potential (4.2) then induces an effective potential for φ :

$$V_{\rm eff}(\varphi) = \mu l_s^{(n+d)} e^{\tilde{n}\varphi/m_{pl}}, \qquad (4.7)$$

where $\tilde{n} = (n-d)/\sqrt{2d}$. Note that the original bulk potential needs to be multiplied by the area of the orbifold fixed plane in order to obtain the effective potential for φ , $V_{\text{eff}}(\varphi)$. There is also a factor of b^{-2d} coming from converting to the Einstein frame (see e.g. [25], appendix A). The equation of motion for φ then becomes

$$\ddot{\varphi} + 3H\dot{\varphi} = -\tilde{n}\frac{\mu l_s^{(n+d)}}{m_{pl}}e^{\tilde{n}\varphi/m_{pl}}$$
(4.8)

with

$$H^{2} = \frac{1}{3m_{pl}^{2}} \left[\frac{\dot{\varphi}^{2}}{2} + \frac{\mu l_{s}^{(n+d)}}{m_{pl}} e^{\tilde{n}\varphi/m_{pl}} \right]$$
(4.9)

During Phase 2, the scale factor a(t) of the three spatial dimensions parallel to the orbifold fixed planes will expand according to the usual four space-time dimensional cosmological equations, where matter is dominated by the scalar field φ . The solution of the equations of motion (4.8 and 4.9) in the cases $\tilde{n} = 1$ and $\tilde{n} = 2$ is given by

$$\varphi = \frac{m_{pl}}{\tilde{n}} \ln \frac{2m_{pl}^2(6-\tilde{n}^2)}{\tilde{n}^4 \mu l_s^{(n+d)} t^2}$$
(4.10)

The corresponding values of the equation of state parameter are

$$w = \frac{\tilde{n}^2 - 3}{3}.$$
 (4.11)

For $\tilde{n} = 1$ this equation of state corresponds to an accelerating background, but for $\tilde{n} = 2$ the background evolution is non-accelerating. In fact, as \tilde{n} grows one can easily show that the usual inflationary slow-roll conditions are grossly violated. Thus, for a value of $\tilde{n} \ge 2$ or equivalently $n \ge 2\sqrt{2d} + d$ the evolution of a(t) during this phase will be non-inflationary.

Taking into account the bulk expansion during Phase 1 of (4.1), it follows that for d = 6and p = 3 the initial value of φ is about $69m_{pl}$. The exponential form of the potential will lead to a rapid collapse of the extra dimensions. To estimate the time scale of the decrease, we replace the source of the right hand side of (4.8) by its initial value and estimate the time interval Δt for φ to decrease by an amount m_{pl} . We find that this time interval equals the initial Hubble time. Thus, a rough estimate of the duration of Period 2 is $10^2 H^{-1}$.

5. Modulus stabilization and late time cosmology

The next crucial step in our scenario is to invoke a mechanism to stabilize the radius of the extra dimensions at a fixed radius. Such modulus stabilization mechanisms have recently been extensively studied both in the context of string theory models of inflation (see e.g. [37] for recent reviews) and in string gas cosmology [36]. We will make use of the mechanism developed in the latter approach.

String modes which carry momentum about the extra dimensions will generate an effective potential for the radion which is repulsive. These repulsive effects will dominate for values of the radion smaller than the self-dual radius. Since these modes are very light at large values of the radion, it is likely that they will be present in great abundance. Even if they are not, the subset of such modes which are massless at enhanced symmetry points will be copiously produced when the value of the radion approaches such points [30, 31].⁴ The induced potential will lead to a source term in the equation of motion for the scale factor b(t) which is of the form [32, 33]

$$\ddot{b} + 3H\dot{b} = 8\pi Gn(t) \left[\left(\frac{1}{b}\right)^2 - b^2 \right] + \cdots, \qquad (5.1)$$

 $^{^{4}}$ As discussed in [33], stabilization via string modes which are massless at the self-dual radius leads to a consistent late time cosmology.

where the dots indicate extra source terms from other string modes, as well as terms quadratic in \dot{b} . Note that n(t) is given by the number density of the modes. Translating to the scalar field φ , and neglecting terms quadratic in $\dot{\varphi}$, the above equation becomes

$$\ddot{\varphi} + 3H\dot{\varphi} = 8\pi Gn(t)e^{-\varphi/m_{pl}}m_{pl}\left(e^{-2\varphi/m_{pl}} - e^{2\varphi/m_{pl}}\right).$$
(5.2)

Thus, it follows that after approaching the self-dual radius, b(t) will perform damped oscillations about b(t) = 1, or, in other words, $\varphi(t)$ will undergo damped oscillations about and get trapped at $\varphi = 0$ (which corresponds to string scale separation between the orbifold fixed planes). At this separation, the four dimensional effective potential V_{eff} becomes

$$V_{\text{eff}} = \Lambda^{d+4+n} l_s^{n+d}, \tag{5.3}$$

and, taking upper limit on Λ from (4.4), this becomes

$$V_{\rm eff} = l_{\rm s}^{-4} 10^{-30(d-p+3+n)} \,. \tag{5.4}$$

Thus, starting with vanishing cosmological constant in the bare bulk lagrangian, our scenario accidentely generates a cosmological constant energy density in our present universe which is suppressed by $30 \times (d - p + 3 + n)$ orders of magnitude. This will provide the correct order of the cosmological constant to account for the current acceleration if d = pand n = 1.

Either at some point during the phase of contraction, or else when the distance between the orbifold fixed planes has decreased to the string scale, all of the bulk branes will decay, presumably predominantly into radiation along the fixed plane directions. The three unconfined spatial dimensions will thus emerge in the expanding radiation-dominated phase of standard cosmology. The energy density which at late time governs the dynamics of our scale factor a(t) is the bulk energy density integrated over the transverse dimensions. Since the bulk energy in Phase 1 is dominated by the p = 3 branes, the integrated energy density is constant. Thus, at the beginning of the radiation-dominated phase the effective energy density is of the same order of magnitude as the initial bulk energy density, namely given by the string scale.

From the point of view of late time cosmology, what has been achieved during Phase 1 is to increase the size of our spatial sections without decreasing the effective energy density. Without extra spatial dimensions, the energy density can only remain constant if the expansion of space is inflationary, but making use of the dynamics of extra spatial dimensions, constant effective energy density can be achieved using non-accelerated expansion of all dimensions.

Note that in the case of p > 3, specifically in the cases where we use Neveu-Schwarz 5-branes in the bulk, the projected energy density actually increases in Phase 1. If it decreases less during Phase 2 than it increased during Phase 1 (which will be the case e.g. if the branes convert to radiation during Phase 2), then the possibility emerges that we are driven to a Hagedorn phase of string theory towards the end of Phase 2 [22, 38]. In this case, a very nice mechanism for the generation of a scale-invariant spectrum of fluctuations [26] can be realized. This possibility will be briefly discussed in the next section.

There is another key prediction of our model which is closely related to the chosen topology of space. No odd-dimensional cycles exist on the inner space T^6/Z_2 , thus prohibiting certain stable configurations of p-branes. Given that we are using odd-dimensional branes in our examples, only 1 or 3 brane dimensions can wrap our three-dimensional toroidal space T^3 , because no odd-dimensional stable p-branes can have an odd number of their brane dimensions wrapped about the inner space. This prevents the creation of stable "stringy" domain walls and monopoles in our universe, but it may predict the existence and future detection of cosmic strings.

6. Discussion and conclusions

By making use of some tools coming from string theory, we have proposed a mechanism to solve the entropy (size) problem of standard cosmology without inflation. According to our proposal, the universe begins hot, small and dense. We assume that the six extra spatial dimensions of perturbative superstring theory are orbifolded, the three dimensions we see today are not (they are toroidal). The universe emerges with a gas of bulk branes (e.g. three branes if we have the perturbative limit of Type IIB superstring theory in mind or 5-branes if we start from heterotic string theory or M-theory) which drives an initial phase of isotropic bulk expansion of all nine spatial dimensions. During this phase, the energy density projected onto the orbifold fixed planes does not decrease, even though the scale factor is expanding (as $t^{1/3}$ in the case of 3-branes in six extra dimensions). We assume the presence of a weak confining potential between the orbifold fixed planes (the cosmological scenario which emerges when considering a more conventional type of potential will be discussed in a followup paper [39]). Such a potential will eventually dominate over the bulk energy density and will lead to a second phase in which the extra spatial dimensions rapidly contract while our three spatial dimensions continue to expand. Once the orbifold fixed planes approach each other to within the string scale, stringy effects previously studied in the context of string gas cosmology will stabilize the radion degrees of freedom. The bulk branes decay, and the universe emerges into the radiation-dominated phase of standard cosmology, with a temperature which is of string scale, but a size which is many orders of magnitude larger than what would be expected on dimensional arguments.⁵ Note that in the proposed scenario, the electroweak scale and the late time acceleration may be explained by the same source.

Since the initial spatial section is in thermal contact, the horizon problem of standard cosmology is explained, as well. Our scenario, however, does not solve the flatness problem of standard cosmology. If the initial spatial sections are curved, then the curvature will lead to a re-collapse of the universe. One way to address the flatness problem is to invoke a special symmetry such as the BPS symmetry (see e.g. [41] for a textbook discussion) which prohibits spatial curvature.

⁵Note that our proposal has certain similarities with the approach of [40], in which — in the context of brane world cosmology — it was proposed that the decay of Kaluza-Klein bulk modes will lead to an entropy flow from the bulk to the brane which can solve the entropy and homogeneity problem of standard cosmology without requiring a phase of inflationary expansion.

In order to provide an alternative to inflation in terms of solving all of the cosmological problems of standard cosmology which inflation addresses, we need to find a mechanism for generating fluctuations. Work on this topic is in progress. Since the universe is initially in causal contact, there are no causality arguments which prevent the generation of adiabatic fluctuations. It is possible that bulk fluctuations similar to the ones proposed in the Ekpyrotic scenario could play this role. Provided there are scale-invariant fluctuations in bulk metric variables during the contracting phase, the work of [42] (see also [43]) shows that such fluctuations will induce a scale-invariant spectrum of four dimensional metric fluctuations in the radiation-dominated phase. Another possibility, in particular in the context of branes with spatial dimension larger than three, is that the post-collapse phase will lead to such high densities that a quasi-static Hagedorn phase will result. The Hagedorn phase makes a smooth transition to the radiation-dominated phase of standard cosmology. In this case, string thermodynamics automatically generates a scale-invariant spectrum of adiabatic fluctuations on all scales smaller than the Hubble radius during the quasi-static phase [26].

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